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## COMPARING PURE YANG-MILLS $SU(2)$ AND $SU(3)$ PROPAGATORS

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The infrared behavior of gluon and ghost propagators in Yang-Mills gauge theories is of central importance for the understanding of confinement in QCD. While analytic studies using Schwinger-Dyson equations predict the same infrared exponents for the  $SU(2)$  and  $SU(3)$  gauge groups, lattice simulations usually assume that the two cases are different, although their qualitative infrared features may be the same. We carry out a comparative study of lattice (Landau) propagators for both gauge groups. Our data were especially produced with equivalent lattice parameters to allow a careful comparison of the two cases.

### 1. Introduction and Motivation

The study of the infrared limit of QCD is of central importance for the comprehension of the mechanisms of quark and gluon confinement and of chiral-symmetry breaking. In Landau gauge, the Gribov-Zwanziger<sup>1</sup> and the Kugo-Ojima<sup>2</sup> confinement scenarios predict, at small momenta, an enhanced ghost propagator and a suppression of the gluon propagator. The strong infrared divergence for the ghost propagator corresponds to a long-range interaction in real space, which may be related to quark confinement. The suppression of the gluon propagator, which should vanish at zero momentum, implies (maximal) violation of reflection positivity which may be viewed as an indication of gluon confinement. Analytic studies of gluon and ghost propagators using Schwinger-Dyson equations (SDE) seem to agree with the above scenarios.<sup>3</sup> In particular, they predict for the gluon and for the ghost propagators an infrared exponent that is independent of the gauge group  $SU(N_c)$ .

The Landau-gauge gluon propagator  $D(k^2)$  and ghost propagator  $G(k^2)$  have been studied by several groups in quenched QCD [i.e. pure  $SU(3)$  Yang-Mills the-

ory], in pure  $SU(2)$  Yang-Mills theory (in 2, 3 and 4 space-time dimensions) and in full QCD.<sup>4,5,6,7,8,9,10,11,12,13</sup> In all cases the ghost propagator is enhanced when compared to the tree-level behavior  $1/k^2$ . On the other hand, lattice studies suggest a finite nonzero infrared gluon propagator, in contradiction with the infrared Schwinger-Dyson solution. One should note that finite-size effects are large in the gluon case, as is suggested by investigation of SDE on a 4-torus<sup>14</sup>, and difficult to evaluate. Nevertheless, violation of reflection positivity is confirmed by several numerical studies in 3d and in 4d, for the  $SU(2)$  and the  $SU(3)$  gauge groups.<sup>12</sup>

When comparing gluon and ghost propagators from SDE studies to numerical results, the agreement is usually at the qualitative level. Moreover, while analytic studies using Schwinger-Dyson equations predict the same infrared exponents for the  $SU(2)$  and  $SU(3)$  gauge groups, lattice simulations usually assume that the two cases are different, although their qualitative infrared features may be the same. In this paper, we carry out a comparative study of lattice Landau gauge propagators for these two gauge groups. Our data were especially produced by considering equivalent lattice parameters in order to allow a careful comparison of the two cases.

## 2. Numerical Simulations and Results

We consider four different sets of lattice parameters, with the same lattice size  $N^4$  and the same physical lattice spacing  $a$  for the two gauge groups. In particular, in the  $SU(3)$  case we used  $N^4 = 16^4$  at  $\beta = 6.0$ ,  $N^4 = 24^4$  at  $\beta = 6.2$ ,  $N^4 = 32^4$  at  $\beta = 6.4$  and  $N^4 = 32^4$  at  $\beta = 6.0$ . For these cases the lattice spacing was taken from Ref. 15. The corresponding  $\beta$  values for  $SU(2)$  were computed using the asymptotic scaling analysis discussed in Ref. 13, yielding  $\beta = 2.4469, 2.5501$  and  $2.6408$ . In the first three cases the physical lattice volume  $V = (Na)^4$  is approximately the same, i.e.  $V \approx (1.7 \text{ fm})^4$ . The fourth case corresponds to a significantly larger physical volume, i.e.  $V \approx (3.2 \text{ fm})^4$ .

The gluon propagator  $D(k)$  and the ghost propagator  $G(k)$  were evaluated as a function of the lattice momentum  $k$ , using 50 configurations for all four cases. In order to compare the propagators from the different simulations, the propagators were renormalized to their tree-level value  $1/\mu^2$ , using  $\mu = 3 \text{ GeV}$  as a renormalization point. Details of the simulations can be found in Ref. 16.

In what concerns finite-volume and finite-lattice-spacing effects, we find a slight dependence on the type of momenta  $[(k, 0, 0, 0), (k, k, 0, 0), (k, k, k, 0)$  or  $(k, k, k, k)]$  where the renormalization is performed. Fig. 1 illustrates the effects of choosing different renormalization constants for the gluon propagator. From now on, we will consider only the data computed using renormalization constants for momenta  $(k, 0, 0, 0)$ .

In Fig. 2 we show the ratios of the renormalized  $SU(3)$  over  $SU(2)$  propagators for various lattice setups. The data show that the two cases have very similar finite-size and discretization effects. Moreover,  $SU(2)$  and  $SU(3)$  propagators are

essentially equal, with slight differences in the low-momenta region (especially for the gluon propagator). Thus, our study supports the prediction from the Schwinger-Dyson equations that the propagators are the same for all  $SU(N_c)$  groups in the nonperturbative region. Clearly, further studies are required before drawing conclusions about the comparison between  $SU(2)$  and  $SU(3)$  propagators in the deep-infrared region, where the gluon propagator may show a turnover and a suppression, as predicted in the Gribov-Zwanziger scenario.

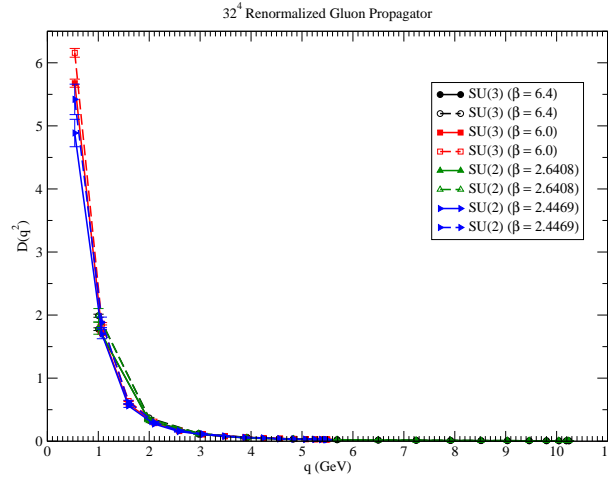


Fig. 1. Renormalized gluon propagator for  $32^4$  lattices. The solid line is the propagator computed using the renormalization constant  $Z$  associated with momenta  $(k, 0, 0, 0)$ . The dashed line uses  $Z$  associated with  $(k, k, 0, 0)$  momenta. In this figure, the data report only momenta of  $(k, k, 0, 0)$  type.

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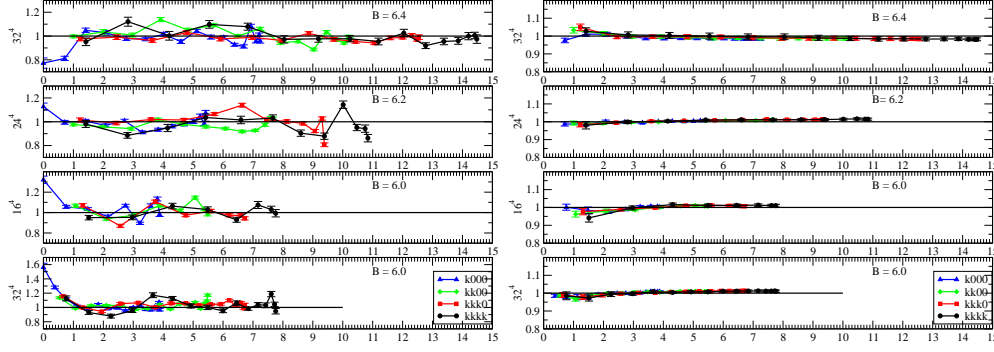


Fig. 2. Ratios of  $SU(3)$  over  $SU(2)$  gluon (left) and ghost (right) propagators (for the four lattice setups) considered as a function of the magnitude  $k$  of the four momentum in GeV.

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